

SHARP OFF-DIAGONAL WEIGHTED WEAK TYPE ESTIMATES FOR SPARSE OPERATORS

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Abstract. We prove sharp weak type weighted estimates for a class of sparse operators that includes majorants of standard singular integrals, fractional integral operators, and square functions. These bounds are known to be sharp in many cases, and our main new result is the optimal bound

$$[w, \sigma]_{A_{p,q}^\alpha}^{1/q} [w^q]_{A_\infty}^{1/v - 1/p} \lesssim [w]_{A_{p,q}}^{1/q} [w]_{A_{p,q}}^{1/v - 1/p} = [w]_{A_{p,q}}^{1/v - \frac{\alpha}{q}}$$

for $p > v$ and Sobolev type condition $\frac{1}{q} + \frac{\alpha}{d} = \frac{1}{p}$. For $v \leq q \leq \frac{v}{1 - \frac{v\alpha}{d}}$, we also obtain the bounds $[w]_{A_{p,q}}^{1/q}$ and it has an additional logarithmic factor, taking the form $(1 + \log[w^q]_{A_\infty})^{1/v}$. Moreover, we study a class of sparse maximal operators and give the weak type off-diagonal two-weight sharp bound.

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