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ON THE ERDŐS–LAX INEQUALITY CONCERNING POLYNOMIALS

GRADIMIR V. MILOVANOVIĆ AND ABDULLAH MIR

Abstract. If $P(z)$ is a polynomial of degree n which does not vanish in $|z| < k$, where $k \leqslant 1$, then N. K. Govil [On a theorem of S. Bernstein, *Proc. Nat. Acad. Sci.*, **50** (1980), 50–52] proved that

$$\max_{|z|=1} |P'(z)| \leqslant \frac{n}{1+k^n} \max_{|z|=1} |P(z)|,$$

provided $|P'(z)|$ and $|Q'(z)|$ attain maximum at the same point on $|z| = 1$, where $Q(z) = z^n \overline{P(1/\overline{z})}$. In this paper, we obtain certain refinements and generalizations of this inequality and related results.

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