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GEODESIC SANDWICH THEOREM WITH AN APPLICATION

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Abstract. The main goal of the paper is to prove the sandwich theorem for geodesic convex functions in a complete Riemannian manifold. Then by using this theorem we have proved an inequality in a manifold with bounded sectional curvature. Finally, we have shown that the gradient of a convex function is orthogonal to the tangent vector at some point of any geodesic.

Mathematics subject classification (2010): 26B25, 39B62, 52A20, 52A30, 52A41, 53C22.
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REFERENCES

[1] D. AZAGRA, J. FERRERA, AND F. LOPEZ-MESAS, *Nonsmooth analysis and Hamilton-Jacobi equations on Riemannian manifolds*, J. Funct. Anal., **220**, 2 (2005), 304–361.
[2] K. BARON, J. MATKOWSKI, AND K. NIKODEM, *A sandwich with convexity*, Math. Pannonica, **5**, 2 (1994), 139–144.
[3] R. L. BISHOP, AND B. O'NEILL, *Manifolds of negative curvature*, Trans. Amer. Math. Soc., **145** (1969), 1–49.
[4] F. C. MITROI-SYMEONIDIS, *A sandwich theorem for convex set valued functions*, An. Univ. Oradea Fasc. Mat., **1** (2016), 77–79.
[5] R. E. GREENE, AND H. WU. H., *On the subharmonicity and plurisubharmonicity of a geodesic convex function*, Indiana Univ. Math. J., **22**, 7 (1973), 641–653.
[6] A. IQBAL, I. AHMAD, AND S. ALI, S. *Some properties of geodesic semi-E-convex functions*, Nonlinear Anal., **74**, 17 (2011), 6805–6813.
[7] A. IQBAL, S. ALI, AND I. AHMAD, I., *On geodesic E-convex sets, geodesic E-convex functions and E-epigraphs*, J. Optim. Theory Appl., **155**, 1 (2012), 239–251.
[8] J. JOST, *Riemannian geometry and geometric analysis*, Springer, Berlin, 2011.
[9] T. LARA, J. MATKOWSKI, N. MERENTES, AND R. QUINTERO, *A Generalization of m-Convexity and a Sandwich Theorem*, Ann. Math. Sil., **31**, 1 (2017), 107–126.
[10] J. MATKOWSKI, AND M. WRÒBEL, *Sandwich theorem for m-convex functions*, J. Math. Anal. Appl., **451** (2017), 924–930.
[11] K. NIKODEM, AND S. WASOWICZ, *A sandwich theorem and Hyers-Ulam stability of affine functions*, Aequationes Math., **49**, 1 (1995), 160–164.
[12] T. RAPCSAK, *Smooth nonlinear optimization in* $\mathbb{R}^n$, Kluwer Academic Publisher, 1997.
[13] R. SCHOEN, AND S. T. YAU, *Lectures on differential geometry. Conference Proceedings and Lecture Notes in Geometry and Topology*, I. International Press, Cambridge, MA, 1994.
[14] A. A. SHAIKH, A. IQBAL, AND C. K. MONDAL, *Some results on* φ*-convex functions and geodesic* φ*-convex functions*, Differ. Geom. Dyn. Syst. **20** (2018), 159–170.
[15] C. UDRIŞTE, *Convex functions and optimization methods on Riemannian manifolds*, Kluwer Academic Publisher, 1994.

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