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WEIGHTED WEAK–TYPE INEQUALITIES FOR SQUARE FUNCTIONS

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Abstract. The paper is devoted to weighted weak-type inequalities for square functions of continuous-path martingales and identifies the optimal dependence of the weak norm on the characteristic of the weight. The proof rests on Bellman function technique: the estimates are deduced from the existence of special functions enjoying appropriate size conditions and concavity.

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REFERENCES

[1] R. BAÑUELOS AND A. OSĘKOWSKI, *Weighted norm inequalities for fractional maximal operators - a Bellman function approach*, Indiana Math. J. 64 (2015), 957–972.
[2] R. BAÑUELOS AND A. OSĘKOWSKI, *Sharp Weighted L^2 inequalities for square functions*, Trans. Amer. Math. Soc. 370 (2018), pp. 2391–2422.
[3] R. BAÑUELOS AND A. OSĘKOWSKI, *Weighted square function estimates*, to appear in Bull. Sci. Math.
[4] S. M. BUCKLEY, *Estimates for operator norms on weighted spaces and reverse Jensen inequalities*, Trans. Amer. Math. Soc. **340** (1993), 253–272.
[5] C. DELLACHERIE AND P.-A. MEYER, *Probabilities and potential B: Theory of martingales*, North Holland, Amsterdam, 1982.
[6] C. DOMINGO-SALAZAR, M. LACEY AND G. REY, *Borderline weak-type estimates for singular integrals and square functions*, Bull. London Math. Soc. 48 (2016), 63–73.
[7] T. HYTÖNEN, *The sharp weighted bound for general Calderón-Zygmund operators*, Ann. Math. 175 (2012), 1473–1506.
[8] T. HYTÖNEN AND K. LI, *Weak and strong A_p-A_∞ estimates for square functions and related operators*, Proc. Amer. Math. Soc. 146 (2018), 2497–2507.
[9] M. IZUMISAWA AND N. KAZAMAKI, *Weighted norm inequalities for martingales*, Tôhoku Math. Journ. **29** (1977), 115–124.
[10] M. T. LACEY, K. MOEN, C. PÉREZ AND R. H. TORRES, *Sharp weighted bounds for fractional integral operators*, J. Funct. Anal. **259** (2010), pp. 1073–1097.
[11] A. LERNER, *Sharp weighted norm inequalities for Littlewood-Paley operators and singular integrals*, Adv. Math., 226 (2011), 3912–3926.
[12] B. MUCKENHOUPT, *Weighted norm inequalities for the Hardy maximal function*, Trans. Amer. Math. Soc **165** (1972), 207–226.
[13] F. NAZAROV, A. REZNIKOV, S. TREIL AND A. VOLBERG, *A Bellman function proof of the L^2 bump conjecture*, J. Anal. Math. **121** (2013), 255–277.
[14] F. NAZAROV AND S. TREIL, *The hunt for a Bellman function: applications to estimates for singular integral operators and to other classical problems of harmonic analysis*, St. Petersburg Math. J. **8** (1997), 721–824.
[15] F. NAZAROV, S. TREIL AND A. VOLBERG, *The Bellman functions and two-weight inequalities for Haar multipliers*, J. Amer. Math. Soc., **12** (1999), pp. 909–928.
[16] A. OSĘKOWSKI, *Weighted Inequalities for the Dyadic Square Function*, Integral Equations and Operator Theory 85 (2016), pp. 359–380.

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[17] A. OSĘKOWSKI, *Weighted inequalities for martingale transforms and stochastic integrals*, Mathematika 63 (2017), 433–450.
[18] G. PESKIR, *A Change-of-Variable Formula with Local Time on Surfaces*, In: Donati-Martin C., Émery M., Rouault A., Stricker C. (eds) Séminaire de Probabilités XL. Lecture Notes in Mathematics, vol 1899. Springer, Berlin, Heidelberg, 2007.
[19] V. VASYUNIN, *The exact constant in the inverse Hölder inequality for Muckenhoupt weights (Russian)*, Algebra i Analiz **15** (2003), 73–117; translation in St. Petersburg Math. J. **15** (2004), pp. 49–79.
[20] G. WANG, *Differential subordination and strong differential subordination for continuous-time martingales and related sharp inequalities*, Ann. Probab. 23 (1995), no. 2, 522–551.
[21] J. WITTWER, *A sharp bound for the martingale transform*, Math. Res. Lett. **7** (2000), 1–12.