

## OPTIMAL $L^p$ HARDY–RELLICH TYPE INEQUALITIES ON THE SPHERE

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**Abstract.** In this paper we study some  $L^p$ -Hardy-Rellich type inequalities and the corresponding optimal constant on the geodesic sphere. By the divergence theorem, properties of radial Laplacian and geodesic distance, we obtain an improved version of Hardy-Rellich inequalities holding in dimension  $N \geq 3$ . The result is new for  $N = 3, 4$ . Moreover, we show that the constant obtained is optimally sharp.

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