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MAXIMAL VALUES OF SYMMETRIC FUNCTIONS IN DISTANCES BETWEEN POINTS

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Abstract. In this note we find the maximal values of several symmetric functions in the variables which are the squares of distances $|z_i - z_j|^2$, $1 \leqslant i < j \leqslant d$, between some d complex points $z_1, \dots, z_d$ in the unit disc. We compute the maximums of σ_m, for $m = 1,2,3,4$, explicitly and find the conditions on $z_1, \dots, z_d$ under which those maximal values are attained. This problem is motivated by an inequality of Cassels (1966) and a subsequent conjecture of Alexander.

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