Mathematical
Inequalities
& Applications
Volume 23, Number 1 (2020), 341–350
doi:10.7153/mia-2020-23-26

ON JAMES TYPE CONSTANTS AND THE NORMAL STRUCTURE IN BANACH SPACES

ZHAN-FEI ZUO

Abstract. In this paper, we establish the lower bounds for the weakly convergent sequence coefficient $WCS(X)$ of a Banach space X, in terms of the James type constant $J_{X,t}(\tau)$, the coefficient of weak orthogonality $\mu(X)$ and Domínguez-Benavides coefficient $R(1,X)$. By mean of these bounds, we identify some geometrical properties implying normal structure. Meanwhile, the James type constant $J_{X,t}(\tau)$, the coefficient of weak orthogonality $\mu(X)$ and Domínguez-Benavides coefficient $R(1,X)$ for the Bynum space $l_{2,\infty}$ are computed to show that our estimates are sharp.

Mathematics subject classification (2010): 46B20, 46B25.
Keywords and phrases: James type constant, weakly convergent sequence coefficient, normal structure.

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