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ON THE AREAS OF MIDPOINT POLYGONS

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Abstract. For a polygon $V_1...V_n$ in the Euclidean plane, let $V_1^1...V_n^1$ denote its midpoint polygon. By induction, its m-th midpoint polygon $V_1^m...V_n^m$ is defined to be the midpoint polygon of $V_1^{m-1}...V_n^{m-1}$. In this paper, we will give different kinds of formulas of the area of $V_1^m...V_n^m$. We will describe the limit behavior of the area as m goes to infinity, and we will determine the infimum and the supremum of the area among all convex $V_1...V_n$ with a fixed area. Some affine invariants derived from the area will also be discussed.

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