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WEIGHTED ESTIMATES FOR ROUGH SINGULAR INTEGRALS WITH APPLICATIONS TO ANGULAR INTEGRABILITY, II

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Abstract. This paper is devoted to studying certain singular integral operators with rough radial kernel h and sphere kernel Ω as well as the corresponding maximal operators along polynomial curves. The authors establish several weighted estimates for such operators by assuming that the kernels $h \equiv 1$ and $\Omega \in \mathscr{F}_\beta(\mathrm{S}^{n-1})$, or $h \in \Delta_\gamma(\mathbb{R}_+)$ and $\Omega \in W\mathscr{F}_\beta(\mathrm{S}^{n-1})$. Here $\mathscr{F}_\beta(\mathrm{S}^{n-1})$ denotes the Grafakos-Stefanov kernel and $W\mathscr{F}_\beta(\mathrm{S}^{n-1})$ denotes the variant of Grafakos-Stefanov kernel. As applications, the boundedness of such operators on the mixed radial-angular spaces $L^p_{|x|}L^q_\theta(\mathbb{R}^n)$ are obtained. Meanwhile, the corresponding vector-valued versions are also given. Moreover, the bounds are independent of the coefficients of the polynomials in the definition of operators.

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