CHARACTERIZATION OF INNER PRODUCT SPACES AND QUADRATIC FUNCTIONS BY SOME CLASSES OF FUNCTIONS

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Abstract. We define and discuss the c-quadratic-midaffine and F-midaffine functions. Using these functions we characterize inner product spaces and quadratic functions, respectively.

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