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ON SOME TRACE INEQUALITIES

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Abstract. In this paper we consider some generalizations of the Ando inequality

$$|||f(A)-f(B)||| \leqslant |||f(|A-B|)|||$$

with the "weight" $(A-B)^p$. More precisely, for $p \geqslant 1$ such that $(-1)^p = -1$ and for a non-negative function f on $[0,\infty)$ such that $f(0)=0$, we study the following inequality:

$$\mathrm{Tr}\,((A-B)^p(f(A)-f(B))) \geqslant \mathrm{Tr}\,(|A-B|^p f(|A-B|)),$$

whenever A and B are positive semidefinite matrices. We show that the inequality is true for any operator convex function f and it is reversed whenever f is operator monotone.

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REFERENCES

[1] ANDO, T., *Comparison of norms* $|||f(A)-f(B)|||$ *and* $|||f(|A-B|)|||$, Math. Z. **197**(3), 403–410 (1988).
[2] AUDENAERT, K. M. R., CALSAMIGLIA, J., MUÑOZ TAPIA, R., BAGAN, E., MASANES, LL., ACIN, A., VERSTRAETE, F., *Discriminating states: The quantum Chernoff bound*, Phys. Rev. Lett. **98**, 160501 (2007).
[3] AUDENAERT, K. M. R., *Trace inequalities for completely monotone functions and Bernstein functions*, Linear Algebra Appl. **437**(2), 601–611 (2012).
[4] BHATIA, R., DAVIS, C., *More matrix forms of the arithmetic-geometric mean inequality*, SIAM J. Matrix Anal. Appl. **14**(1), 132–136 (1993).
[5] BOURIN, J. C., *Matrix versions of some classical inequalities*, Linear Algebra Appl. **416**(2-3), 890–907 (2006).
[6] BOURIN, J. C., LEE, E. Y., *Matrix inequalities from a two variables functional*, Internat. J. Math. **27**(09), 1650071 (2016).
[7] CARLEN, E., *Trace inequalities and quantum entropy: An introductory course*, Contemporary mathematics **529**, 73–140 (2010).
[8] DINH, T. H., HO, M. T., LE, C. T., VO, B. K., *Two trace inequalities for operator functions*, to appear in Mathematical Inequalities and Applications (MIA), 2019.
[9] ZHAN, X., *Matrix Inequalities*, Lecture Notes in Mathematics, Springer-Berlin-Heidelberg, 2004.
[10] RICARD, É., *An inequality in noncommutative* L^p *-spaces*, J. Math. Anal. Appl. **438**(1), 53–58 (2016).

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