INEQUALITIES FOR ANGLES BETWEEN SUBSPACES WITH APPLICATIONS TO CAUCHY–SCHWARZ INEQUALITY IN INNER PRODUCT SPACES

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Abstract. We show several inequalities for angles between vectors and subspaces in inner product spaces, where concave functions are involved. In specific situations, some of them can be interpreted as triangle inequalities for natural metrics on complex projective spaces. In a consequence, we obtain a few operator generalizations of the famous Cauchy-Schwarz inequality, where powers grater than two occur.

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