A NEW IMPROVED FORM OF THE HILBERT
INEQUALITY AND ITS APPLICATIONS

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Abstract. In this paper, it shows a new improved form of the Hilbert inequality by introducing a proper weight function $\Omega(\lambda, x)$ with a parameter $\lambda (\lambda > \frac{1}{2})$. As applications, a new refinement of Widder's inequality and an extension of Hardy-Littlewood's inequality are given.

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