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SHARP ESTIMATES FOR THE ZEROS OF THE DERIVATIVE OF OSCILLATING POLYNOMIALS WITH LAGUERRE WEIGHT

LOZKO MILEV AND NIKOLA NAIDENOV

Abstract. Denote by $\mathscr{V}_n(\lambda)$ the set of all weighted polynomials of the form $f(x)=e^{-\lambda x}p(x)$ $(\lambda>0)$, where p is an algebraic polynomial of degree n which has n simple real zeros. Given $f\in\mathscr{V}_n(\lambda)$, let $x_1<\cdots<x_n$ and $t_1<\cdots<t_n$ be the zeros of f and f', correspondingly. Set $h_k:=x_{k+1}-x_k$, $k=1,\dots,n-1$. We prove sharp estimates of the forms

$$x_k+c_kh_k\leqslant t_k\leqslant x_{k+1}-d_kh_k,\quad k=1,\dots,n-1,$$

and

$$x_n+c_nh_{n-1}\leqslant t_n\leqslant x_n+d_nh_{n-1},$$

with explicit expressions for the coefficients, depending on λ. Known estimates of the same type for algebraic polynomials can be obtained by letting $\lambda\to 0$.

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