

## FRACTIONAL DIFFERENTIAL OPERATORS IN VECTOR-VALUED SPACES AND APPLICATIONS

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*Abstract.* Fractional differential operator equations with parameter are studied. Uniform  $L_p$ -separability properties and sharp resolvent estimates are obtained for elliptic equations in terms of fractional derivatives. Moreover, maximal regularity properties of the fractional abstract parabolic equation are established. Particularly, it is proven that the operators generated by these equations are positive and also are generators of analytic semigroups. As an application, the anisotropic parameter dependent fractional differential equations and the system of fractional differential equations are studied.

*Mathematics subject classification (2010):* 35R11, 35J25, 35K90, 47F05, 43A95.

*Keywords and phrases:* Fractional-differential equations, Sobolev-Lions spaces, differential-operator equations, maximal  $L_p$  regularity, abstract parabolic equations, operator-valued multipliers.

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