

LOWER BOUNDS FOR THE FIRST EIGENVALUES OF THE p -LAPLACIAN AND THE WEIGHTED p -LAPLACIAN

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Abstract. In this paper, we investigate the p -Laplacian Δ_p on a complete noncompact submanifold of a Riemannian manifold with sectional curvature bounded above by a negative constant. Moreover, we study the weighted p -Laplacian $\Delta_{p,\varphi}$ on an n -dimensional complete noncompact smooth metric measure space $(M, g, e^{-\varphi} dv)$ with M being a submanifold in the hyperbolic space $\mathbb{H}^m(-1)$. We obtain some estimates for their first eigenvalues. They reflect the relations between the first eigenvalues of these two kinds of nonlinear operators and the geometrical data of manifolds. Our results cover a result derived by Lin (Nonlinear Anal., **148** (2017), 126–137) for the Laplacian, some results of Du and Mao (J. Math. Anal. Appl., **456** (2017), 787–795) for the drifting Laplacian and the p -Laplacian.

Mathematics subject classification (2010): 35P30, 47J10, 53C21.

Keywords and phrases: Eigenvalue, p -Laplacian, weighted p -Laplacian, hyperbolic space, metric measure space.

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