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ON THE ITERATED MEAN TRANSFORMS OF OPERATORS

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Abstract. Let $T = U|T|$ be the polar decomposition of an operator $T \in \mathscr{L}(\mathscr{H})$. For given $s,t \geqslant 0$, we say that $\widehat{T}_{s,t} := sU|T| + t|T|U$ is the weighted mean transform of T. In this paper, we study properties of the k-th iterated weighted mean transform $\widehat{T}^{(k)}_{s,t}$ of $T = U|T|$ when U is unitary. In particular, we give the polar decomposition of such $\widehat{T}^{(k)}_{s,t}$ and investigate its applications. Finally, we consider the iterated weighted mean transforms of a weighted shift.

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