

OPERATOR NORM AND NUMERICAL RADIUS ANALOGUES OF COHEN'S INEQUALITY

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Abstract. Let D be an invertible multiplication operator on $L^2(X, \mu)$, and let A be a bounded operator on $L^2(X, \mu)$. In this note we prove that $\|A\|^2 \leq \|DA\| \|D^{-1}A\|$, where $\|\cdot\|$ denotes the operator norm. If, in addition, the operators A and D are positive, we also have $w(A)^2 \leq w(DA)w(D^{-1}A)$, where w denotes the numerical radius.

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