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NORM INEQUALITIES OF DAVIDSON–POWER TYPE

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Abstract. Let A,B, and X be $n\times n$ complex matrices such that A and B are positive semidefinite. It is shown, among other inequalities, that

$$\|AX+XB\| \leqslant \frac{1}{2}\max(\|A\|,\|XBX^*\|)+\frac{1}{2}\max(\|X^*AX\|,\|B\|)+\left\|A^{1/2}XB^{1/2}\right\|.$$

This norm inequality extends an inequality of Kittaneh, which improves an earlier inequality of Davidson and Power.

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REFERENCES

[1] R. BHATIA, *Matrix Analysis*, Springer, New York, 1997.

[2] R. BHATIA AND F. KITTANEH, *On the singular values of a product of operators*, SIAM J. Matrix Anal. Appl. **11** (1990), 272–277.

[3] J.-C. BOURIN, *Convexity or concavity inequalities for Hermitian operators*, Math. Ineq. Appl. **7** (2004), 607–620.

[4] J.-C. BOURIN, *A matrix subadditivity inequality for symmetric norms*, Proc. Amer. Math. Soc. **138** (2009), 495–504.

[5] J.-C. BOURIN AND A. MHANNA, *Positive block matrices and numerical ranges*, C.R. Acad. Sci. Paris **355**(2017), 1077–1081.

[6] K. DAVIDSON, S. C. POWER, *Best approximation in C^*-algebras*, J. Reine Angew. Math. **368** (1986), 43–62.

[7] R. A. HORN AND C. R. JOHNSON, *Matrix Analysis*, 2nd Edition, Cambridge University Press, 2013.

[8] J. C. HOU AND H. K. DU, *Norm inequalities of positive operator matrices*, Integral Equations Operator Theory **22**(1995), 281–294.

[9] F. KITTANEH, *Norm inequalities for certain operator sums*, J. Funct. Anal. **143** (1997), 337–348.

[10] F. KITTANEH, *Norm inequalities for sums of positive operators*, J. Operator Theory **48** (2002), 95–103.

[11] F. KITTANEH, *Inequalities for commutators of positive operators*, J. Funct. Anal. **250** (2007), 132–143.

[12] X. ZHAN, *Singular values of differences of positive semidefinite matrices*, SIAM J. Matrix Anal. Appl. **22** (2002), 819–823.