ON EXTREMALS FOR THE TRUDINGER–MOSER INEQUALITY
WITH VANISHING WEIGHT IN THE N–DIMENSIONAL UNIT BALL

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Abstract. In this paper, we study the extremal function for the Trudinger-Moser inequality with vanishing weight in the unit ball $B \subset \mathbb{R}^N$ ($N \geq 3$). To be exact, let $\mathcal{S}$ be the set of all decreasing radially symmetrical functions and $\alpha_N = N \omega_{N-1}^{1/(N-1)}$, where $\omega_{N-1}$ is the area of the unit sphere in $\mathbb{R}^N$. Suppose $h$ is a nonnegative radially symmetrical function belonging to $C^0(B)$ satisfying $h(x) > 0$ in $B \setminus \{0\}$ and $h(x)|x|^{-N\beta} \to 1$ as $x \to 0$ for some real number $\beta \geq 0$. By means of blow-up analysis, we prove that the supremum

$$\Lambda_\beta := \sup_{u \in W^{1, N}_0(B) \cap \mathcal{S}, \|\nabla u\|_N \leq 1} \int_B \exp \left\{ \alpha_N (1 + \beta) \|u\|_N^{N-1} \right\} h(x) \, dx$$

can be attained by some $u_0 \in W^{1, N}_0(B) \cap \mathcal{S}$ with $\|\nabla u_0\|_N = 1$. This improves a recent result of Yang-Zhu [39].


Keywords and phrases: Trudinger-Moser inequality, blow-up analysis, extremal function.

REFERENCES


