

AN INEQUALITY INVOLVING A TRIANGLE AND AN INTERIOR POINT AND ITS APPLICATION

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Abstract. Let \mathbf{x}_0 be an interior split point in the triangle $T := [\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3]$. By α_{ij} we denote the angle $\widehat{\mathbf{x}_0, \mathbf{x}_i, \mathbf{x}_j}$, $i \neq j$. We show that

$$\cos \alpha_{12} \cos \alpha_{23} \cos \alpha_{31} + \cos \alpha_{21} \cos \alpha_{32} \cos \alpha_{13} > 0.$$

Additionally, we use this inequality to prove uniqueness and existence of a conforming quadratic piecewise harmonic finite element on the Clough-Tocher split of a triangle.

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REFERENCES

- [1] P. ALFELD, AND T. SOROKINA, *Linear Differential Operators on Bivariate Spline Spaces and Spline Vector Fields*, BIT Numerical Mathematics, **56**, 1(2016), 15–32.
- [2] S. C. BRENNER AND L. R. SCOTT, *The mathematical theory of finite element methods*, Third edition, Texts in Applied Mathematics, **15**, Springer, New York, 2008.
- [3] M.-J. LAI AND L. L. SCHUMAKER, *Spline functions on triangulations*, Cambridge University Press, Cambridge, 2007.
- [4] D. S. MITRINOVIĆ, J. E. PEČARIĆ AND V. VOLENEC, *Recent Advances in geometric Inequalities*, Kluwer Academic Publishers, The Netherlands, 1989.
- [5] S. ZHANG AND T. SOROKINA, *Conforming harmonic finite elements on the Hsieh-Clough-Tocher split of a triangle*, submitted.