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SHARP BERNSTEIN INEQUALITIES USING CONVEX ANALYSIS TECHNIQUES

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Abstract. In this paper we consider the space of polynomials of degree at most three in the real line endowed with the sup norm over the unit interval. We provide, explicitly, all the extreme points of the unit ball of this space. Using the previous geometrical description, we obtain the Bernstein function for the first and second derivative of the polynomials of degree at most 3.

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