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AN INEQUALITY FOR THE ANALYSIS OF VARIANCE

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Abstract. We prove a generalization to matrices and tensors of the Szőkefalvi-Nagy inequality and the Grüss-Popoviciu inequality. Our more general version is required in the analysis of variance (ANOVA).

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