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SOME HARDY AND CARLESON MEASURE SPACES ESTIMATES FOR BOCHNER–RIESZ MEANS

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Abstract. In this paper, we show that the Bochner-Riesz means are bounded on weighted and variable Hardy spaces by using the finite atomic decomposition theories. The boundedness of Bochner-Riesz means on weighted and variable Carleson measure spaces is also obtained. Moreover, we also prove that the maximal Bochner-Riesz means are bounded from weighted or vaiable Hardy spaces to weighted or variable Lebesgue spaces.

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