Mathematical
Inequalities
& Applications
Volume 23, Number 3 (2020), 1051–1077
doi:10.7153/mia-2020-23-81

# ON STEVIĆ–SHARMA OPERATORS FROM WEIGHTED BERGMAN SPACES TO WEIGHTED–TYPE SPACES

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*Abstract.* Let $\mathscr{H}(\mathbb{D})$ be the space of analytic functions on the unit disc $\mathbb{D}$. Let $\varphi$ be an analytic self-map of $\mathbb{D}$ and $\psi_1, \psi_2 \in \mathscr{H}(\mathbb{D})$. Let $C_\varphi$, $M_\psi$ and $\mathscr{D}$ denote the composition, multiplication and differentiation operators, respectively. In order to treat the products of these operators in a unified manner, Stević et al. introduced the following operator

$$T_{\psi_1,\psi_2,\varphi} f = \psi_1 \cdot f \circ \varphi + \psi_2 \cdot f' \circ \varphi, \qquad f \in \mathscr{H}(\mathbb{D}).$$

We characterize the boundedness and compactness of the operators $T_{\psi_1,\psi_2,\varphi}$ from weighted Bergman spaces to weighted-type and little weighted-type spaces of analytic functions. Also, we give examples of bounded, unbounded, compact and non compact operators $T_{\psi_1,\psi_2,\varphi}$.

*Mathematics subject classification* (2010): 47B33, 47B38.

*Keywords and phrases*: Differentiation operators, multiplication operators, composition operators, weighted composition operators, weighted-type spaces of analytic functions, weighted Bergman spaces.

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