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A NEW GENERALIZED REFINEMENT OF THE WEIGHTED ARITHMETIC–GEOMETRIC MEAN INEQUALITY

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Abstract. In this paper, we prove that for $i = 1,2,\dots,n$, $a_i \geqslant 0$ and $\alpha_i > 0$ satisfy $\sum_{i=1}^n \alpha_i = 1$, then for $m = 1,2,3,\dots$, we have

$$\left(\prod_{i=1}^n a_i^{\alpha_i}\right)^m + r_0^m\left(\sum_{i=1}^n a_i^m - n\sqrt[n]{\prod_{i=1}^n a_i^m}\right) \leqslant \left(\sum_{i=1}^n \alpha_i a_i\right)^m$$

where $r_0 = \min\{\alpha_i : i = 1,\dots,n\}$. This is a considerable generalization of the two refinements of the arithmetic-geometric mean inequality due to Furuichi [2], Manasrah and Kittaneh [7], which correspond to the cases $m = 1$ and $n = 2$, respectively. As application we give some generalized inequalities of determinants for positive definite matrices.

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