A NEW GENERALIZED REFINEMENT OF THE WEIGHTED ARITHMETIC–GEOMETRIC MEAN INEQUALITY

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Abstract. In this paper, we prove that for \( i = 1, 2, \ldots, n \), \( a_i \geq 0 \) and \( \alpha_i > 0 \) satisfy \( \sum_{i=1}^{n} \alpha_i = 1 \), then for \( m = 1, 2, 3, \ldots \), we have

\[
\left( \prod_{i=1}^{n} a_i^{\alpha_i} \right)^m + r_0^m \left( \sum_{i=1}^{n} a_i^m - n \sqrt[2n]{\prod_{i=1}^{n} a_i^m} \right) \leq \left( \sum_{i=1}^{n} \alpha_i a_i \right)^m
\]

where \( r_0 = \min\{ \alpha_i : i = 1, \ldots, n \} \). This is a considerable generalization of the two refinements of the arithmetic-geometric mean inequality due to Furuichi [2], Manasrah and Kittaneh [7], which correspond to the cases \( m = 1 \) and \( n = 2 \), respectively. As application we give some generalized inequalities of determinants for positive definite matrices.


Keywords and phrases: Arithmetic-geometric mean inequality, Young inequality.

REFERENCES