Mathematical
Inequalities
& Applications
Volume 23, Number 3 (2020), 1117–1125
doi:10.7153/mia-2020-23-85

CAUCHY—SCHWARZ TYPE INEQUALITIES AND APPLICATIONS TO NUMERICAL RADIUS INEQUALITIES

FUAD KITTANEH AND HAMID REZA MORADI

Abstract. We present new improvements of certain Cauchy–Schwarz type inequalities. As applications of the results obtained, we provide refinements of some numerical radius inequalities for Hilbert space operators. It is shown, among other inequalities, that if $A \in \mathbb{B}(\mathscr{H})$, then

$$\omega^2(A) \leqslant \frac{1}{6}\left\||A|^2 + |A^*|^2\right\| + \frac{1}{3}\omega(A)\,\||A| + |A^*|\|.$$

Mathematics subject classification (2010): 47A12, 47A30, 15A60.
Keywords and phrases: Cauchy–Schwarz inequality, numerical radius, operator norm, mixed Schwarz inequality.

REFERENCES

[1] J. AUJLA AND F. SILVA, *Weak majorization inequalities and convex functions*, Linear Algebra Appl. **369** (2003), 217–233.
[2] P. BHUNIA, S. BAG, AND K. PAUL, *Numerical radius inequalities and its applications in estimation of zeros of polynomials*, Linear Algebra Appl. **573** (2019), 166–177.
[3] M. L. BUZANO, *Generalizzazione della diseguaglianza di Cauchy–Schwarz, (Italian)*, Rend. Sem. Mat. Univ. e Politech. Torino. **31** (1971/73), 405–409 (1974).
[4] S. S. DRAGOMIR, *Power inequalities for the numerical radius of a product of two operators in Hilbert spaces*, Sarajevo J. Math. **5**(18) (2009), 269–278.
[5] T. FURUTA, J. MIĆIĆ, J. PEČARIĆ, AND Y. SEO, *Mond–Pečarić method in operator inequalities*, Element, Zagreb, 2005.
[6] P.R. HALMOS, *A Hilbert space problem book*, 2nd ed., Springer, New York, 1982.
[7] F. KITTANEH, *A numerical radius inequality and an estimate for the numerical radius of the Frobenius companion matrix*, Studia Math. **158**(1) (2003), 11–17.
[8] F. KITTANEH, *Numerical radius inequalities for Hilbert space operators*, Studia Math. **168**(1) (2005), 73–80.
[9] M. E. OMIDVAR, H. R. MORADI, AND K. SHEBRAWI, *Sharpening some classical numerical radius inequalities*, Oper. Matrices. **12**(2) (2018), 407–416.

Paper MIA-23-85