Mathematical
Inequalities
& Applications
Volume 23, Number 3 (2020), 1135–1143 doi:10.7153/mia-2020-23-87

FURTHER INTERPOLATION INEQUALITIES RELATED TO ARITHMETIC–GEOMETRIC MEAN, CAUCHY–SCHWARZ AND HÖLDER INEQUALITIES FOR UNITARILY INVARIANT NORMS

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Abstract. An inequality for matrices that interpolates between the Cauchy-Schwarz and the arithmetic-geometric mean inequalities for unitarily invariant norms has been obtained by Audenaert. Alakhrass obtained a related result to Audenaert's inequality using a log-convex function g defined on $[0,1]$. Very recently, Zou obtained an inequality for matrices that unifies Hölder's inequality and the arithmetic-geometric mean inequality for unitarily invariant norms. A generalized version of Zou's inequality for unitarily invariant norms is given, and an alternative proof of Audenaert's inequality using a refined version of Alakhrass's function is presented.

Mathematics subject classification (2010): 15A18, 15A60, 47A30.

Keywords and phrases: Singular value, unitarily invariant norm, inequality, interpolation.

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