Mathematical
Inequalities
& Applications
Volume 23, Number 4 (2020), 1179–1195
doi:10.7153/mia-2020-23-91

WEAK TYPE ENDPOINT ESTIMATES FOR THE COMMUTATORS OF ROUGH SINGULAR INTEGRAL OPERATORS

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Abstract. Let Ω be homogeneous of degree zero and have mean value zero on the unit sphere S^{n-1}, T_Ω be the convolution singular integral operator with kernel $\frac{\Omega(x)}{|x|^n}$. For $b \in \mathrm{BMO}(\mathbb{R}^n)$, let $T_{\Omega,b}$ be the commutator of T_Ω. In this paper, by establishing suitable sparse dominations, the authors establish some weak type endpoint estimates of $L\log L$ type for $T_{\Omega,b}$ when $\Omega \in L^q(S^{n-1})$ for some $q \in (1,\infty]$.

Mathematics subject classification (2010): 42B20, 47A30.
Keywords and phrases: Commutator, singular integral operator, sparse operator, maximal operator.

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