

AN INEQUALITY INVOLVING THE CONSTANT e AND A GENERALIZED CARLEMAN–TYPE INEQUALITY

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Abstract. In this paper, we establish a double inequality involving the constant e . As an application, we give a generalized Carleman-type inequality.

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REFERENCES

- [1] H. ALZER AND C. BERG, *Some classes of completely monotonic functions*, Ann. Acad. Sci. Fenn. Math. **27** (2002), 445–460.
- [2] H. J. BROTHERS AND J. A. KNOX, *New closed-form approximations to the logarithmic constant e* , Math. Intelligencer **20** (1998), 25–29.
- [3] T. CARLEMAN, *Sur les fonctions quasi-analytiques*, Comptes rendus du V^e Congrès des Mathématiciens Scandinaves, Helsingfors (1922), 181–196.
- [4] H.-W. CHEN, *On an infinite series for $(1 + 1/x)^x$ and its application*, Int. J. Math. Math. Sci. **29** (2002), 675–680.
- [5] C.-P. CHEN, *Generalization of weighted Carleman-type inequality*, East J. Approx. **12** (2006), 63–69.
- [6] C.-P. CHEN, W.-S. CHEUNG AND F. QI, *Note on weighted Carleman type inequality*, Int. J. Math. Math. Sci. **3** (2005), 475–481.
- [7] C.-P. CHEN AND J. CHOI, *Asymptotic formula for $(1 + 1/x)^x$ based on the partition function*, Amer. Math. Monthly **121** (2014), 338–343.
- [8] C.-P. CHEN, N. ELEZOVIĆ AND L. VUKŠIĆ, *Asymptotic formulae associated with the Wallis power function and digamma function*, J. Classical Anal. **2**, 2 (2013), 151–166.
- [9] C.-P. CHEN AND F. QI, *Generalization of Hardy’s inequality*, Proc. Jangjeon Math. Soc. **7** (2004), 57–61.
- [10] A. ČIŽMEŠIJA, J. PEČARIĆ AND L. E. PERSSON, *On strengthened weighted Carleman’s inequality*, Bull. Austral. Math. Soc. **68** (2003), 481–490.
- [11] S. S. DRAGOMIR AND Y.-H. KIM, *The strengthened Hardy inequalities and their new generalizations*, Fac. Sci. Math. **20**, 2 (2006), 39–49.
- [12] G. H. HARDY, *Notes on some points in the integral calculus*, Messenger of Math. **54** (1925), 150–156.
- [13] G. H. HARDY, J. E. LITTLEWOOD AND G. PÓLYA, *Inequalities*, 2nd ed., Cambridge Univ. Press, Cambridge, 1952.
- [14] Y. HU, *A strengthened Carleman’s inequality*, Commun. Math. Anal. **1** (2006) 115–119.
- [15] M. JOHANSSON, L. E. PERSSON AND A. WEDESTIG, *Carleman’s inequality—history, proofs and some new generalizations*, J. Inequal. Pure Appl. Math. **4**, 3 (2003), Art. 53.
http://www.emis.de/journals/JIPAM/images/135_02_JIPAM/135_02_www.pdf.
- [16] S. KAIJSER, L. E. PERSSON AND A. ÖBERG, *On Carleman and Knopp’s inequalities*, J. Approx. Theory **117** (2002), 140–151.
- [17] J. A. KNOX AND H. J. BROTHERS, *Novel series-based approximations to e* , College Math. J. **30** (1999), 269–275.
- [18] A. KUFNER AND L. E. PERSSON, *Weighted Inequalities of Hardy Type*, World Scientific, New Jersey/London, Singapore/Hong Kong, 2003.
- [19] J.-L. LI, *Notes on an inequality involving the constant e* , J. Math. Anal. Appl. **250** (2000), 722–725.

- [20] H.-P. LIU AND L. ZHU, *New strengthened Carleman's inequality and Hardy's inequality*, J. Inequal. Appl. J. Inequal. Appl. **2007**, Art. ID 84104, 7 pp.
<http://link.springer.com/article/10.1155/2007/84104/fulltext.html>.
- [21] Z. LÜ, Y. GAO AND Y. WEI, *Note on the Carleman's inequality and Hardy's inequality*, Comput. Math. Appl. **59** (2010), 94–97.
- [22] C. MORTICI AND Y. HU, *On some convergences to the constant e and improvements of Carleman's inequality*, Carpathian J. Math. **31** (2015), 249–254.
- [23] C. MORTICI AND X.-J. JANG, *Estimates of $(1+x)^{1/x}$ Involved in Carleman's Inequality and Keller's Limit*, Filomat **29**, 7 (2015), 1535–1539.
- [24] J. PEČARIĆ AND K. B. STOLARSKY, *Carleman's inequality: history and new generalizations*, Aequationes Math. **61** (2001), 49–62.
- [25] G. PÓLYA, *Proof of an inequality*, Proc. London Math. Soc. **24** (1926), 57.
- [26] Z. XIE AND Y. ZHONG, *A best approximation for constant e and an improvement to Hardy's inequality*, J. Math. Anal. Appl. **252** (2000), 994–998.
- [27] P. YAN AND G.-Z. SUN, *A strengthened Carleman's inequality*, J. Math. Anal. Appl. **240** (1999), 290–293.
- [28] B.-C. YANG AND L. DEBNATH, *Some inequalities involving the constant e , and an application to Carleman's inequality*, J. Math. Anal. Appl. **223** (1998), 347–353.
- [29] B.-C. YANG, *On Hardy's inequality*, J. Math. Anal. Appl. **234** (1999), 717–722.
- [30] X.-J. YANG, *On Carleman's inequality*, J. Math. Anal. Appl. **253** (2001), 691–694.
- [31] X.-J. YANG, *Approximations for constant e and their applications*, J. Math. Anal. Appl. **262** (2001), 651–659.