

BILINEAR WEIGHTED HARDY-TYPE INEQUALITIES IN DISCRETE AND q -CALCULUS FRAMEWORKS

PANKAJ JAIN, SAIKAT KANJILAL, GULDARYA E. SHAMBILOVA AND
VLADIMIR D. STEPANOV

Abstract. We characterize Hardy inequality in weighted Lebesgue sequence spaces involving discrete bilinear Hardy operator $\left(\sum_{i=-\infty}^n a_i\right)\left(\sum_{i=-\infty}^n b_i\right)$ and then we apply this information to characterize the inequality with q -bilinear Hardy operator

$$\mathcal{H}_q(f, g)(x) := \left(\int_0^{\infty} \chi_{(0, x]}(t) f(t) d_q t\right) \left(\int_0^{\infty} \chi_{(0, x]}(t) g(t) d_q t\right)$$

for all possible indices of summation.

Mathematics subject classification (2010): 26D10, 46E35.

Keywords and phrases: Weighted Hardy inequality, bilinear Hardy-type inequalities, discrete Hardy inequality, q -calculus, Jackson integral.

REFERENCES

- [1] A.O. BAIARYSTANOV, L.E. PERSSON, S. SHAIMARDAN AND A. TEMIRKHANOVA, *Some new Hardy-type inequalities in q -analysis*, J. Math. Inequal. **10** (2016), 761–781.
- [2] G. BENNETT, *Some elementary inequalities*, Quart. J. Math. Oxford Ser. (2), **38** (1987), 401–425.
- [3] G. BENNETT, *Some elementary inequalities II*, Quart. J. Math. Oxford Ser. (2), **39** (1988), 385–400.
- [4] G. BENNETT, *Some elementary inequalities III*, Quart. J. Math. Oxford Ser. (2), **42** (1991), 149–174.
- [5] M. S. BRAVERMAN AND V. D. STEPANOV, *On the discrete Hardy inequality*, Bull. London Math. Soc. **26** (1994), 283–287.
- [6] M.I. AGUILAR CAÑESTRO, P. ORTEGA SALVADOR AND C. RAMÍREZ TORREBLANCA, *Weighted bilinear Hardy inequalities*, J. Math. Anal. Appl. **387** (2012) 320–334.
- [7] P. CHEUNG AND V. KAC, *Quantum calculus*, Edwards Brothers, Inc., Ann Arbor, MI, USA, 2000.
- [8] T. ERNST, *A comprehensive treatment of q -calculus*, Birkhäuser/Springer Basel AG, Basel, 2012.
- [9] T. ERNST, *A new method of q -calculus*, Doctoral thesis, Uppsala university, 2002.
- [10] A. GOGATISHVILI, P. JAIN AND S. KANJILAL, *On bilinear Hardy inequality and corresponding geometric mean inequality*, Ricerche di Mat., <https://doi.org/10.1007/s11587-020-00536-2>.
- [11] K. -G. GROSSE-ERDMANN, *The blocking technique, weighted mean operators and Hardy's inequality*, Lecture Notes in Mathematics, **1679**, (1998), Springer-Verlag: Berlin.
- [12] G. H. HARDY, J.E. LITTLEWOOD AND G. POLYA, *Inequalities*, Reprint of the 1952 edition. Cambridge Mathematical Library, Cambridge University Press, Cambridge, 1988. xii+324 pp.
- [13] F. H. JACKSON, *On q -definite integrals*, Quart. J. Pure Appl. Math. **41** (1910), 193–203.
- [14] S. KANJILAL, L.E. PERSSON AND G. SHAMBILOVA, *Equivalent integral conditions related to bilinear Hardy-type inequalities*, Math. Inequal. Appl., **22:4** (2019), 1535–1548.
- [15] M. KREPELA, *Iterating bilinear Hardy inequalities*, Proc. Edinb. Math. Soc. **60:4** (2017), 955–971.
- [16] A. KUFNER, L. MALIGRANDA AND L. -E. PERSSON, *The Hardy inequality. About its history and some related results*, Vydavatelský Servis, Plzen, 2007. 162 pp.
- [17] A. KUFNER, L.-E. PERSSON AND N. SAMKO, *Weighted Inequalities of Hardy Type*, Second Edition, World Scientific New Jersey, 2017.
- [18] L. MALIGRANDA, R. OINAROV AND L.-E. PERSSON, *On Hardy q -inequalities*, Czechoslovak Math. J., **64:3** (2014), 659–682.

- [19] B. OPIC AND A. KUFNER, *Hardy-Type Inequalities*, Pitman Research Notes in Mathematics Series, **211**, Longman Scientific and Technical, Harlow, 1990.
- [20] D. V. PROKHOROV, V. D. STEPANOV AND E. P. USHAKOVA, *Hardy-Steklov integral operators: Part I*, Proc. Steklov Inst. Math. **300**, Suppl. 2 (2018), S1–S112.
- [21] S. SHAIMARDAN, *Hardy-type inequalities quantum calculus*, Doctoral thesis, Luleå University of Technology, 2018.
- [22] V. D. STEPANOV AND G. E. SHAMBILOVA, *On iterated and bilinear integral Hardy-type operators*, Math. Inequal. Appl., **22**:4 (2019), 1505–1533.
- [23] V. D. STEPANOV AND E. P. USHAKOVA, *Bilinear Hardy-type inequalities in weighted Lebesgue spaces*, Nonlinear Studies, **26**:4 (2019), 939–953.