

## PROOFS OF CONJECTURES OF ELEZOVIĆ AND VUKŠIĆ CONCERNING THE INEQUALITIES FOR MEANS

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*Abstract.* By using the asymptotic expansion method, Elezović and Vukšić conjectured certain inequalities related to Neuman-Sándor mean. The aim of this paper is to offer a proof of these inequalities.

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