GENERALIZED CSISZÁR'S f–DIVERGENCE
FOR LIPSCHITZIAN FUNCTIONS

Đilda Pečarić, Josip Pečarić and Dora Pokaz

Abstract. We started with the generalization of the Csiszár's f-divergence. We stated and proved Jensen's type inequality for L-Lipschitzian functions. The results for commonly used examples of f-divergences, such as the Kullbach-Leibler divergence, the Hellinger divergence, the Rényi divergence and χ^2-distance are derived. Further, we examined two specific averaging functions, previously known in the literature. Finally, we obtained interesting results concerning the Zipf-Mandelbrot law.

Mathematics subject classification (2010): 26D10, 26D15.
Keywords and phrases: f-divergence, Lipschitzian function, Zipf-Mandelbrot law.

REFERENCES

[1] S. M. ALI AND S. D. SILVEY, *A general class of coefficients of divergence of one distribution from another*, J. Roy. Statist. Soc. series B, **28** (1), (1966), 131–142.

[2] R. BERAN, *Minimum Hellinger distance estimates for parametric models*, Ann. Statist., **5** (1977), 445–463.

[3] Y. J. CHO, M. MATIĆ AND J. PEČARIĆ, *Inequalities of Jensen's type for Lipschitzian mappings*, Comm. on Appl. Nonlinear Analysis **8** (2001), 37–46.

[4] Y. J. CHO, M. MATIĆ AND J. PEČARIĆ, *Two mappings in connection to Jensen's inequality*, Panamerican Math. J. **12** (1), (2002), 43–50.

[5] I. CSISZÁR, *Eine Informationstheoretische Ungleichung und ihre Anwendung auf den Bewis der Ergodizität von Markhoffschen Ketten*, Publ. Math. Inst. Hungar. Acad. Sci. **8**, (1963), 85–108.

[6] I. CSISZÁR, *Information-type measures of difference of probability functions and indirect observations*, Studia Sci. Math. Hungar, **2** (1967), 299–318.

[7] S. S. DRAGOMIR, *A mapping in connection to Hadamard's inequalities*, J. Math. Anal. Appl. **167** (1992), 49–56.

[8] S. S. DRAGOMIR, *On Hadamard's inequalities for convex functions*, Mat. Balkanicae **6** (4), (1992), 215–222.

[9] S. S. DRAGOMIR, Y. J. CHO AND S. S. KIM, *Inequalities of Hadamand's type for Lipschitzian mappings and their applications*, J. Math. Anal. Appl. **245** (2000), 489–501.

[10] L. HORVÁTH, G. PEČARIĆ AND J. PEČARIĆ, *Estimations of f- and Rényi divergences by using a cyclic refinement of the Jensen's inequality*, J. Bull. Malays. Math. Sci. Soc. 2017, https://doi.org/10.1007/s40840-017-0526-4.

[11] P. A. KLUZA, M. NIEZGODA, *On Csiszár and Tsallis type f-divergences induced by superquadratic and convex functions*, Math. Inequal. Appl. **21**, 2 (2018), 455–467.

[12] S. KULLBACK, *Information Theory and Statistics*, J. Wiley, New York, 1959.

[13] S. KULLBACK AND R. A. LEIBLER, *On information and sufficiency*, Annals Math. Statist., **12** (1951), 79–86.

[14] M. MATIĆ, C. E. M. PEARCE, J. PEČARIĆ, *Shannon's and related inequalities in information theory*, Survey on classical inequalities, editor Themistocles M. Rassias, Kluwer Academic Publ., 2000, 127–164.

[15] M. MATIĆ, C. E. M. PEARCE, J. PEČARIĆ, *Some comparison theorems for the mean-value characterization of "useful" information measures*, Southeast Asian Bulletin of Mathematics, **23** (1999), 111–116.

[16] T. MORIMOTO, *Markov processes and the H -theorem*, J. Phys. Soc. Jap. **18** (3), (1963), 328–331.

[17] A. RENYI, *On measures of entropy and information*, Proc. Fourth Berkeley Symp. Math. Statist. Prob., Vol 1, University of California Press, Berkeley, 1961.

Mathematical Inequalities & Applications
www.ele-math.com
mia@ele-math.com