COMPOSITION OPERATORS AND CLOSURES OF A CLASS OF MÖBIUS INVARIANT FUNCTION SPACES IN THE BLOCH SPACE

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Abstract. Closures of a class of Möbius invariant function spaces in the Bloch space are investigated in this paper. Moreover, the boundedness and compactness of composition operators from the Bloch space to closures of such Möbius invariant space in the Bloch space are characterized.

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