Mathematical
Inequalities
& Applications
Volume 24, Number 1 (2021), 129–155
doi:10.7153/mia-2021-24-10

IMPROVED FRACTIONAL HARDY INEQUALITIES FOR DUNKL GRADIENT

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Abstract. We prove an improved fractional Hardy inequality in the Dunkl setting for the weighted space $L^p(\mathbb{R}^N, d\mu_k(x))$. Also we prove a similar inequality for half-space.

Mathematics subject classification (2010): Primary 42B35; Secondary 42B37.
Keywords and phrases: Fractional Hardy inequality, Dunkl Laplacian, fractional Sobolev spaces, improved Hardy inequality.

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Paper MIA-24-10