WEIGHTED HELLINGER DISTANCE
AND IN–BETWEENNESS PROPERTY

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Abstract. In this paper we introduce the weighted Hellinger distance for matrices which is an interpolating between the Euclidean distance and the Hellinger distance. We show the equivalence of the weighted Hellinger distance and the Alpha Procrustes distance. As a consequence, we prove that the matrix power mean $\mu_p(t,A,B) = (tA^p + (1-t)B^p)^{1/p}$ satisfies in-betweenness property in the weighted Hellinger and Alpha Procrustes distances.

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