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ON CONSTANTS IN COCONVEX APPROXIMATION OF PERIODIC FUNCTIONS

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Abstract. Let 2π-periodic function $f \in \mathbb{C}$ change its convexity finitely even many times, in the period. We are interested in estimating the degree of approximation of f by trigonometric polynomials which are coconvex with it, namely, polynomials that change their convexity exactly at the points where f does. We list established Jackson-type estimates of such approximation where the constants involved depend on the location of the points of change of convexity and show that this dependence is essential by constructing a counterexample.

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