

MEAN-TYPE MAPPINGS AND INVARIANCE PRINCIPLE

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Abstract. In the finite dimensional case, mean-type mappings, their invariant means, relations between the uniqueness of invariant means and convergence of orbits of the mapping, are considered. In particular it is shown, that the uniqueness of an invariance mean implies the convergence of all orbits. A strongly irregular mean-type mapping is constructed and its unique invariant mean is determined. An application in solving a functional equation is presented.

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