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CHARACTERIZATION OF APPROXIMATELY MONOTONE AND APPROXIMATELY HÖLDER FUNCTIONS

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Abstract. A real valued function f defined on a real open interval I is called Φ-monotone if, for all $x,y \in I$ with $x \leqslant y$ it satisfies

$$f(x) \leqslant f(y) + \Phi(y-x),$$

where $\Phi : [0, \ell(I)[\to \mathbb{R}_+$ is a given nonnegative error function, where $\ell(I)$ denotes the length of the interval I. If f and $-f$ are simultaneously Φ-monotone, then f is said to be a Φ-Hölder function. In the main results of the paper, using the notions of upper and lower interpolations, we establish a characterization for both classes of functions. This allows one to construct Φ-monotone and Φ-Hölder functions from elementary ones, which could be termed the building blocks for those classes. In the second part, we deduce Ostrowski- and Hermite–Hadamard-type inequalities from the Φ-monotonicity and Φ-Hölder properties, and then we verify the sharpness of these implications. We also establish implications in the reversed direction.

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