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A MULTILINEAR RELLICH INEQUALITY

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Abstract. We prove a multilinear variant of the Rellich inequality on the real line. In particular, we establish the weighted inequality

$$\left(\int_a^b w(\delta(x))\Big|\prod_{k=1}^m u_k(x)\Big|^p dx\right)^{1/p} \leqslant C\prod_{k=1}^m \|u_k''\|_{L^{p_k}(a,b)},\ \ u_k \in C_0^2(a,b),\ \ k=1,\dots,m,$$

with a positive function w on $(0,b-a)$, where $-\infty \leqslant a < b \leqslant +\infty$, m is a positive integer, $\delta(x) = \min\{x-a, b-x\}$ is the distance function on (a,b), and $1/p = \sum_{j=1}^m 1/p_j$, $p_j > 1$, $j = 1,\dots,m$. As a corollary we derive the following estimate

$$\left(\int_a^b \Big|\prod_{j=1}^m u_j(x)\Big|^p \delta(x)^{-2mp} dx\right)^{1/p} \leqslant C\prod_{j=1}^m \|u_j''\|_{L^{p_j}(a,b)}.$$

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