

SHARP INEQUALITIES BETWEEN L^p -NORMS FOR THE HIGHER DIMENSIONAL HARDY OPERATOR AND ITS DUAL

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Abstract. We derive the two-sided inequalities between $L^p(X)$ -norms ($1 < p < \infty$) of the higher dimensional Hardy operator and its dual, where the underlying space X is the Heisenberg group \mathbb{H}^n or the Euclidean space \mathbb{R}^n . The interest of main results is that it relates two-sided inequalities with sharp constants which are dimension free. The methodology is completely depending on the rotation method.

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REFERENCES

- [1] M. CHRIST AND L. GRAFAKOS, *Best constants for two nonconvolution inequalities*, Proc. Amer. Math. Soc., **123**, (1995), 1687–1693.
- [2] T. COULHON, D. MÜLLER AND J. ZIENKIEWICZ, *About Riesz transforms on the Heisenberg groups*, Math. Ann., **305**, (1996), 369–379.
- [3] P. DRÁBEK, H. P. HEINIG AND A. KUFNER, *Higher dimensional Hardy inequality*, Internat. Ser. Numer. Math., **123**, (1997), 537–556.
- [4] G. B. FOLLAND AND E. M. STEIN, *Hardy spaces on homogeneous groups*, Princeton, N. J. Princeton University Press, (1982).
- [5] Z. FU, L. GRAFAKOS, S. LU AND F. ZHAO, *Sharp bounds for m -linear Hardy and Hilbert operators*, Houston J. Math., **38**, (2012), 225–244.
- [6] G. H. HARDY, *Notes on some points in the integral calculus, LX. An inequality between integrals*, Messenger of Math., **54**, (1925), 150–156.
- [7] G. H. HARDY, J. E. LITTLEWOOD AND G. PÓLYA, *Inequalities, 2nd ed.*, Cambridge University Press, Cambridge, 1952.
- [8] S. LU, D. YAN AND F. ZHAO, *Sharp bounds for Hardy type operators on higher-dimensional product spaces*, J. Inequal. Appl., **2013**, 2013:148, 11 pp.
- [9] V. I. KOLYADA, *Optimal relationships between L^p -norms for the Hardy operator and its dual*, Ann. Mat. Pura Appl., **193**, (2014), 423–430.
- [10] A. KUFNER AND L. E. PERSSON, *Weighted inequalities of Hardy type*, World Scientific Publishing Company, Singapore, 2003.
- [11] B. OPIC AND A. KUFNER, *Hardy-type inequalities*, Pitman Research Notes in Mathematics Series, vol. 219, Longman Scientific & Technical, New York, Wiley, 1990.
- [12] S. G. SAMKO, *Hardy inequality in the generalized Lebesgue spaces*, Fract. Calc. Appl. Anal., **6** (2003), 355–362.
- [13] G. SINNAMON, *One-dimensional Hardy-type inequalities in many dimensions*, Proc. Roy. Soc. Edinburgh Sect. A, **128**, (1998), 833–848.
- [14] S. THANGAVELU, *Harmonic analysis on the Heisenberg group*, Progress in Mathematics, vol. 159, Boston, MA: Birkhäuser Boston, 1998.
- [15] S. S. VOLOSIVETS, *Weighted Hardy and Cesàro operators on Heisenberg group and their norms*, Integral Transforms Spec. Funct., **28**, (2017), 940–952.

- [16] S. WANG, S. LU AND D. YAN, *Explicit constants for Hardy's inequality with power weight on n -dimensional product spaces*, *Sci. China Math.*, **55**, (2012), 2469–2480.
- [17] Q. WU AND Z. FU, *Sharp estimates for Hardy operators on Heisenberg group*, *Front. Math. China*, **11**, (2016), 155–172.
- [18] F. ZHAO, Z. FU AND S. LU, *Endpoint estimates for n -dimensional Hardy operators and their commutators*, *Sci. China Math.*, **55**, (2012), 1977–1990.