

WEIGHTED COMPOSITION OPERATORS AND THEIR PRODUCTS ON $L^2(\Sigma)$

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Abstract. In this paper, we study the ascent and descent of weighted composition operators on $L^2(\Sigma)$. In addition, we discuss measure theoretic characterizations of some classical properties for products of these type operators.

Mathematics subject classification (2010): Primary 47B20; Secondary 47B38.

Keywords and phrases: Kato operator, conditional expectation, composition operator, multiplication operator, ascent, descent, polar decomposition, closed range.

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