Mathematical
Inequalities
& Applications
Volume 24, Number 2 (2021), 307–321
doi:10.7153/mia-2021-24-22

BOUNDS FOR INDICES OF COINCIDENCE AND ENTROPIES

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Abstract. In this paper we consider a parameterized family of discrete probability distributions and investigate the Rényi,Tsallis, and Shannon entropies associated with them. Lower and upper bounds for these entropies are obtained, improving some results from the literature. The proofs are based on several methods from classical analysis, theory of dual cones, and the stochastic majorization theory. The Rényi and Tsallis entropies are naturally expressed in terms of the index of coincidence. Consequently we study in detail the index of coincidence associated to the corresponding discrete probability distributions. The obtained results lead immediately to properties of the entropies.

Mathematics subject classification (2010): 94A17, 26A51, 26D10, 39B62.
Keywords and phrases: Probability distribution, index of coincidence, bounds for entropies, convex functions, stochastic majorization technique.

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