

DIRECT AND INVERSE APPROXIMATION THEOREMS OF FUNCTIONS IN THE MUSIELAK–ORLICZ TYPE SPACES

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Abstract. In Musilak-Orlicz type spaces \mathcal{S}_M , direct and inverse approximation theorems are obtained in terms of the best approximations of functions and generalized moduli of smoothness. The question of the exact constants in Jackson-type inequalities is studied.

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