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NEW EQUIAFFINE CHARACTERIZATIONS OF THE ELLIPSOIDS RELATED TO AN EQUIAFFINE INTEGRAL INEQUALITY ON HYPEROVALOIDS

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Abstract. In this paper, we study hyperovaloids from the perspective of the equiaffine differential geometry. As the main result, we establish an optimal integral inequality of the hyperovaloids in terms of the normalized affine scalar curvature and the squared norm of the equiaffine Weingarten form. Since the integral inequality becomes an integral equality if and only if the hyperovaloids are equiaffinely equivalent to the ellipsoids, our results give new equiaffine characterizations of the ellipsoids.

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