BEST CONSTANT OF THE CRITICAL HARDY–LERAY INEQUALITY FOR CURL–FREE FIELDS IN TWO DIMENSIONS

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Abstract. In this note, we prove that the best-possible constant of the critical Hardy-Leray inequality for curl-free fields is $1/4$, just the same value as the one for all smooth fields. This fact contrasts sharply with the recent result on the subcritical Hardy-Leray inequality for curl-free fields by the authors [6], and shows the criticality of the inequality.

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