Mathematical
Inequalities
& Applications
Volume 24, Number 2 (2021), 405–420
doi:10.7153/mia-2021-24-28

JOINT NUMERICAL RADIUS OF SPHERICAL ALUTHGE TRANSFORMS OF TUPLES OF HILBERT SPACE OPERATORS

KAIS FEKI AND TAKEAKI YAMAZAKI

Abstract. Let $\mathbf{T} = (T_1, \ldots, T_d)$ be a d-tuple of operators on a complex Hilbert space $\mathscr{H}$. The spherical Aluthge transform of $\mathbf{T}$ is the d-tuple given by $\widehat{\mathbf{T}} := (\sqrt{P}V_1\sqrt{P}, \ldots, \sqrt{P}V_d\sqrt{P})$ where $P := \sqrt{T_1^*T_1 + \ldots + T_d^*T_d}$ and $(V_1, \ldots, V_d)$ is a joint partial isometry such that $T_k = V_kP$ for all $1 \leqslant k \leqslant d$. In this paper, we prove several inequalities involving the joint numerical radius and the joint operator norm of $\widehat{\mathbf{T}}$. Moreover, a characterization of the joint spectral radius of an operator tuple $\mathbf{T}$ via n-th iterated of spherical Aluthge transform is established.

Mathematics subject classification (2010): Primary 47A13; Secondary 47A12, 47A30.
Keywords and phrases: Spherical Aluthge transform, Duggal transform, joint numerical radius, joint spectral radius, joint operator norm.

REFERENCES

[1] A. ALUTHGE, *On p-hyponormal Operators for $0 < p < 1$*, Integral Equations Operator Theory, 13 (1990), 307–315.
[2] T. ANDO, *Aluthge transforms and the convex hull of the spectrum of a Hilbert space operator*, Recent advances in operator theory and its applications, Oper. Theory Adv. Appl. 160 (2005), 21–39.
[3] R. BHATIA AND C. DAVIS, *A Cauchy-Schearz inequality for operators with applications*, Linear Algebra Appl. 223/224 (1995), 119–129.
[4] H. BAKLOUTI AND K. FEKI, *On joint spectral radius of commuting operators in Hilbert spaces*, Linear Algebra Appl. 557 (2018) 455–463.
[5] H. BAKLOUTI, K. FEKI AND O. A. M. SID AHMED, *Joint numerical ranges of operators in semi-Hilbertian spaces*, Linear Algebra Appl. 555 (2018) 266–284.
[6] C. BENHIDA, R. E. CURTO, S. H. LEE AND J. YOON, *Joint spectra of spherical Aluthge transforms of commuting n-tuples of Hilbert space operators*, C. R. Math. Acad. Sci. Paris 357 (2019), 799–802, https://doi.org/10.1016/j.crma.2019.10.003.
[7] J. W. BUNCE, *Models for n-tuples of noncommuting operators*, J. Funct. Anal. 57 (1984), 21–30.
[8] G. CORACH, H. PORTA AND L. RECHT, *An operator inequality*, Linear Algebra Appl., 142 (1990), 153–158.
[9] M. CHŌ, I. B. JUNG AND W. Y. LEE, *On Aluthge Transforms of p-hyponormal Operators*, Integral Equations Operator Theory, 53 (2005), 321–329.
[10] R. CURTO AND J. YOON, *Toral and spherical Aluthge transforms of 2-variable weighted shifts*, C. R. Acad. Sci. Paris 354 (2016), 1200–1204.
[11] R. CURTO AND J. YOON, *Aluthge transforms of 2-variable weighted shifts*, Integral Equations Operator Theory 90 (2018), Paper number 52, 32 pp.
[12] M. CHŌ, M. TAKAGUCHI, *Boundary points of joint numerical ranges*, Pacific J. Math 95 (1981), 27–35.
[13] M. CHŌ, W. ŻELAZKO, *On geometric spectral radius of commuting n-tuples of operators*, Hokkaido Math. J. 21 (1992), 251–258.
[14] R. E. CURTO, *Applications of several complex variables to multiparameter spectral theory. In Surveys of some recent results in operator theory*, Vol. II, volume 192 of Pitman Res. Notes Math. Ser., pages 25–90. Longman Sci. Tech., Harlow, 1988.
[15] A. T. DASH, *Joint numerical range*, Glasnik Mat. 7 (1972), 75–81.

Paper MIA-24-28

[16] K. DYKEMA AND H. SCHULTZ, *Brown measure and iterates of the Aluthge transform for some operators arising from measurable actions*, Trans. Amer. Math. Soc. 361 (2009), 6583–6593.
[17] C. FOIAS, I. JUNG, E. KO, C. PEARCY, *Complete contractivity of maps associated with the Aluthge and Duggal transforma-tions*, Pac. J. Math. 209 (2003) 249–259.
[18] T. FURUTA, *Invitation to linear operators. From matrices to bounded linear operators on a Hilbert space*, Taylor & Francis Group, London, 2001.
[19] K. E. GUSTAFSON, *The Toeplitz-Hausdorff Theorem of linear Operators*, Proc. Amer. Math. Soc. 25 (1970), 203–204.
[20] K. E. GUSTAFSON, D. K. M. RAO, *Numerical Range*, Springer-Verlag, New York, 1997.
[21] S. HILDEBRANDT, *Numerischer Wertebereich und normale Dilatationen*, Acta Sci. Math. (Szeged) 26 (1965), 187–190.
[22] E. HEINZ, *Beiträge zur Störungstheoric der Spektralzerlegung*, Math. Ann. 123 (1951), 415–438.
[23] I. JUNG, E. KO AND C. PEARCY, *Aluthge transform of operators*, Integral Equations Operator Theory 37 (2000), 437–448.
[24] I. B. JUNG, E. KO AND C. PEARCY, *Spectral pictures of Aluthge transforms of operators*, Integral Equations Operator Theory 40 (2001), 52–60.
[25] J. KIM AND J. YOON, *Aluthge transforms and common invariant subspaces for a commuting n-tuple of operators*, Integral Equations Operator Theory 87 (2017) 245–262.
[26] J. KIM AND J. YOON, *Taylor spectra and common invariant subspaces through the Duggal and generalized Aluthge transforms for commuting n-tuples of operators*, J. Operator Theory 81 (2019) 81-105, http://dx.doi.org/10.7900/jot.2017nov27.2210.
[27] S. H. LEE, W. Y. LEE AND J. YOON, *Subnormality of Aluthge transform of weighted shifts*, Integral Equations Operator Theory 72 (2012), 241–251.
[28] C. K. LI, *C-Numerical Ranges and C-Numerical Radii*, Linear and Multilinear Algebra 37 (1994), 51–82.
[29] C. K. LI AND Y. T. POON, *Convexity of the joint numerical range*, SIAM J. Matrix Anal. Appl. 21 (1999), 668–678.
[30] V. MÜLLER, AND A. SOLTYSIAK, *Spectral radius formula for commuting Hilbert space operators*, Studia Math. 103 (1992), 329–333.
[31] G. POPESCU, *Unitary invariants in multivariable operator theory*, Memoirs of the American Mathematical Society, 200 (941), vi+91 pp (2009).
[32] J. G. STAMPFLI AND J. P. WILLIAMS, *Growth conditions and the numerical range in a Banach algebra*, Tôhoku Math. J. 20 (1968) 417–424.
[33] O. TOEPLITZ, *Das algebraische Analogou zu einem satze von fejer*, Math. Zeit, 2 (1918), 187–197.
[34] J. L. TAYLOR, *A joint spectrum for several commuting operators*, J. Funct. Anal. 6 (1970) 172–191.
[35] D. WANG, *Heinz and McIntosh inequalities, Aluthge transformation and the spectral radius*, Math. Inequal. Appl. 6 (2003), 121–124.
[36] T. YAMAZAKI, *On upper and lower bounds for the numerical radius and an equality condition*, Studia Math. 178 (2007), 83–89.
[37] T. YAMAZAKI, *An expression of spectral radius via Aluthge transformation*, Proc. Amer. Math. Soc. 130 (2002), 1131–1137.