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DISCRETE HARDY'S TYPE INEQUALITIES AND STRUCTURE OF DISCRETE CLASS OF WEIGHTS SATISFY REVERSE HÖLDER'S INEQUALITY

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Abstract. In this paper, we will prove a new discrete weighted Hardy's type inequality with different powers. Next, we will apply this inequality to prove that the forward and backward propagation properties (*self-improving properties*) for the general discrete class $\mathscr{B}^{p,q}$ of weights that satisfy the reverse Hölder inequality hold. As special cases, we will deduce the *self-improving properties* of discrete Gehring and Muckenhoupt weights. An example is considered for illustrations.

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