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REMARKS ON THE MONOTONICITY AND CONVEXITY OF JENSEN'S FUNCTION

YANG HUANG, YONGTAO LI* AND JOSIP PEČARIĆ

Abstract. Let $x_1, x_2, \ldots, x_n$ be nonnegative real numbers. The Jensen function of $\{x_i\}_{i=1}^n$ is defined as $J_s(x) = (\sum_{i=1}^n x_i^s)^{1/s}$, also known as the L_p-norm. It is well-known that $J_s(x)$ is decreasing on $s \in (0, +\infty)$. Moreover, Beckenbach [Amer. Math. Monthly, 53 (1946), 501–505] proved further that $J_s(x)$ is a convex function on $s \in (0, +\infty)$. The goal of this note is two-fold. We first revisit the skillful treatment of the proof of Beckenbach, and then we simplify the proof slightly. Additionally, we give a new proof of the convexity of $J_s(x)$ by using the Hölder inequality, our proof is more succinct and short. On the other hand, we investigate a Jensen-type inequality that arised from Fourier analysis by Stein and Weiss. As a byproduct, the Hardy-Littlewood-Póya inequality is also included.

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