

ON THE GENERALIZED VON NEUMANN–JORDAN TYPE CONSTANT FOR SOME CONCRETE BANACH SPACES

ZHAN-FEI ZUO*, LIANG-WEI WANG, YONG-XIANG ZHAO AND YAN-QIU WU

Abstract. In this paper, we investigate the relations involving the generalized von Neumann–Jordan type constant of the absolute normalized norms $\|\cdot\|_\psi$ and $\|\cdot\|_\phi$, where the convex functions ψ and ϕ are comparable. These conclusions which not only contain some previous results, but also give the exact value of the generalized von Neumann–Jordan type constant for some practical examples in the application of geometric theory of Banach spaces.

Mathematics subject classification (2020): Primary 46B20; Secondary 46B25.

Keywords and phrases: The generalized von Neumann–Jordan type constant, Absolute normalized norm, Convex function.

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